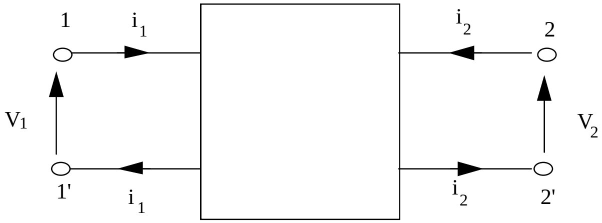
TWO-PORT PARAMETERS

Two-port S-parameters (scattering parameters) are an extremely popular way of representing RF & microwave semiconductor devices and passive components. Here, the AHYZ - parameters used extensively for low frequency circuits are first revised. However, the need for open- and short-circuit terminations to measure these parameters makes them unsuitable for RF & microwave frequencies (low inductance short circuits are hard to realise, and open circuits always have significant capacitance). Furthermore, AHYZ parameters deal with voltages at nodes and currents in branches, whereas high frequency engineers are more likely to deal with voltage waves in transmission-lines (even if the transmission-line is actually a PCB track).



Y-parameters (short Circuit Admittance Parameters)

The Y-parameters of a network are defined in terms of terminal admittances under short circuit conditions

$$Y = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}$$

I=YV is Ohms law in matrix form, and this yields the following:-

$$i_1 = y_{11}v_1 + y_{12}v_2$$

$$i_2 = y_{21}v_1 + y_{22}v_2$$

The Y-parameters can be measured under the conditions of short circuit at either the input or output port. When the output is short circuit $v_2 = 0$ and so :-

$$y_{11} = (i_1/v_1)_{v2=o}, \quad y_{21} = (i_2/v_1)_{v2=o} \quad .$$

y_{11} is therefore defined as the input admittance and y_{21} is defined as the forward transfer admittance with output short circuit. The parameters are referred to as the short circuit input admittance and short circuit forward transfer admittance respectively. When $v_1 = 0$, the input is short circuit and

$$y_{12} = (i_1/v_1)_{v1=o}, \quad y_{21} = (i_2/v_2)_{v1=o}$$

y_{12} is the short circuit reverse transfer admittance. Y_{22} is the short circuit output admittance.

Z-parameters (Open Circuit Impedance Parameters)

$$Z = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix}$$

V = I Z is again Ohm's law in matrix form, yielding:-

$$v_1 = z_{11}i_1 + z_{12}i_2$$

$$v_2 = z_{21}i_1 + z_{22}i_2$$

These can be reduced by open-circuiting either the input or the output. The Z-parameters are then measured as follows:-

$$z_{11} = (v_1/i_1)_{i2=o}, \quad z_{12} = (v_1/i_2)_{i1=o}$$

$$z_{21} = (v_2/i_1)_{i2=o}, \quad z_{22} = (v_2/i_2)_{i1=o}$$

Therefore,

z_{11} is the open circuit input impedance

z_{12} is the open circuit reverse transfer impedance,

z_{21} is the open circuit forward transfer impedance, and

z_{22} is the open circuit output impedance.

A-parameters (Transmission or Chain Parameters)

A-parameters are designed to represent the relationship between the input variables (v1, i1) and the output (v2, i2). In matrix form, we write:-

$$\begin{bmatrix} v_1 \\ i_1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} v_2 \\ -i_2 \end{bmatrix}$$

which yields:-

$$v_1 = a_{11}v_2 - a_{12}i_2$$

$$i_1 = a_{21}v_2 - a_{22}i_2 ,$$

The reason for using $-i_2$ is that it simplifies things when two-ports are connected in cascade, but this convention is not always followed.

The A-parameters are defined by the following expressions

$a_{11} = (v_1/v_2)_{i2=o}$, $a_{12} = (v_1/-i_2)_{v2=o}$

$a_{21} = (i_1/v_2)_{i2=o}$, $a_{22} = (i_1/-i_2)_{v2=o}$

In words,

a_{11} is the inverse of the open circuit forward voltage gain,

a_{12} is the inverse of the short circuit forward transfer admittance,

a_{21} is the inverse of the open circuit forward transfer impedance, and

a_{22} is the inverse of the short circuit forward current gain.

H-parameters (Hybrid parameters)

Although its not immediately obvious why its useful, these relate (v1, i2) to (i1, v2):-

$$H = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}$$ and then:-

$v_1 = h_{11}i_1 + h_{12}v_2$

$i_2 = h_{21}i_1 + h_{22}v_2$,

The H-parameters can be calculated or measured as follows:-

$h_{11} = (v_1/i_1)_{v2=o}$, $h_{12} = (v_1/v_2)_{i1=o}$,

$h_{21} = (i_2/i_1)_{v2=o}$, $h_{22} = (i_2/v_2)_{i1=o}$

In words,

h_{11} is the short circuit input impedance

h_{12} is the open circuit reverse voltage gain,

h_{21} is the short circuit forward current gain, and

h_{22} is the open circuit output admittance.

It is found that H-parameters have a special relevance to bipolar transistors, and are commonly used at RF frequencies. For example, h_{21} is the transistor current gain, and by measuring it the Ft is easily found (the frequency at which the current gain is unity).

G-parameters (Inverse Hybrid Parameters) also exist if you want more.

SUMMARY OF AHYZ PARAMETERS

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

'$[y]$'

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

'$[z]$'

$$\begin{bmatrix} v_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ v_2 \end{bmatrix}$$

'$[h]$'

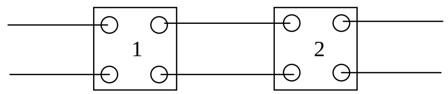
$$\begin{bmatrix} v_1 \\ i_1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} v_2 \\ i_2 \end{bmatrix}$$

'$[a]$'

Interconnection of two-port networks

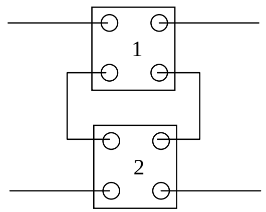
It is when two-ports are connected together in different ways that the usefulness of all these different parameters becomes apparent.

'Cascade':-



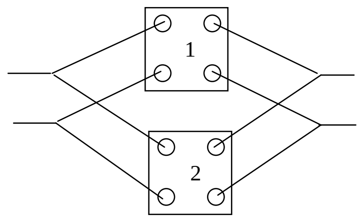
$[a] = [a_1] \times [a_2]$

'Series':-



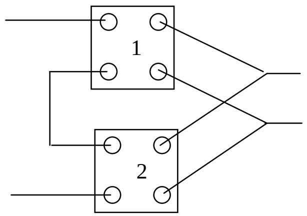
$[z] = [z_1] + [z_2]$

'Parallel':-



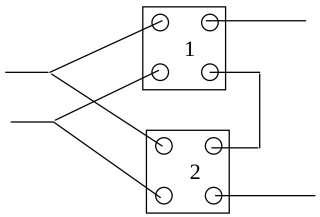
$[y] = [y_1] + [y_2]$

Inputs in series; outputs in parallel:-



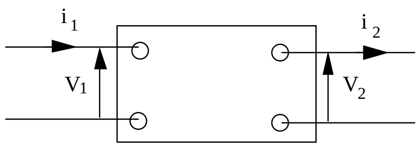
$[h] = [h_1] + [h_2]$

Inputs in parallel; outputs in series:-



$[h]^{-1} = [h_1]^{-1} + [h_2]^{-1}$

Remember that sign change for the ABCD parameters: this is required to make the cascade operation simpler:-



Matrix parameter conversion table

	z		**y**		**h**		**a**	
z	$\mathbf{z_{11}}$	$\mathbf{z_{12}}$	$\frac{y_{22}}{\Delta_y}$	$\frac{-y_{12}}{\Delta_y}$	$\frac{\Delta_h}{h_{22}}$	$\frac{h_{12}}{h_{22}}$	$\frac{a_{11}}{a_{21}}$	$\frac{\Delta_a}{a_{21}}$
	$\mathbf{z_{21}}$	$\mathbf{z_{22}}$	$\frac{-y_{21}}{\Delta_y}$	$\frac{y_{11}}{\Delta_y}$	$\frac{-h_{21}}{h_{22}}$	$\frac{1}{h_{22}}$	$\frac{1}{a_{21}}$	$\frac{a_{22}}{a_{21}}$
y	$\frac{z_{22}}{\Delta_z}$	$\frac{-z_{12}}{\Delta_z}$	$\mathbf{y_{11}}$	$\mathbf{y_{12}}$	$\frac{1}{h_{11}}$	$\frac{-h_{12}}{h_{11}}$	$\frac{a_{22}}{a_{12}}$	$\frac{-\Delta_a}{a_{12}}$
	$\frac{-z_{21}}{\Delta_z}$	$\frac{z_{11}}{\Delta_z}$	$\mathbf{y_{21}}$	$\mathbf{y_{22}}$	$\frac{h_{21}}{h_{11}}$	$\frac{\Delta_h}{h_{11}}$	$\frac{-1}{a_{12}}$	$\frac{a_{11}}{a_{12}}$
h	$\frac{\Delta_z}{z_{22}}$	$\frac{z_{12}}{z_{22}}$	$\frac{1}{y_{11}}$	$\frac{-y_{12}}{y_{11}}$	$\mathbf{h_{11}}$	$\mathbf{h_{12}}$	$\frac{a_{12}}{a_{22}}$	$\frac{\Delta_a}{a_{22}}$
	$\frac{-z_{21}}{z_{22}}$	$\frac{1}{z_{22}}$	$\frac{y_{21}}{y_{11}}$	$\frac{\Delta_y}{y_{11}}$	$\mathbf{h_{21}}$	$\mathbf{h_{22}}$	$\frac{-1}{a_{22}}$	$\frac{a_{21}}{a_{22}}$
a	$\frac{z_{11}}{z_{21}}$	$\frac{\Delta_z}{z_{21}}$	$\frac{-y_{22}}{y_{21}}$	$\frac{-1}{y_{21}}$	$\frac{-\Delta_h}{h_{21}}$	$\frac{-h_{11}}{h_{21}}$	$\mathbf{a_{11}}$	$\mathbf{a_{12}}$
	$\frac{1}{z_{21}}$	$\frac{z_{22}}{z_{21}}$	$\frac{-\Delta_y}{y_{21}}$	$\frac{-y_{11}}{y_{21}}$	$\frac{-h_{22}}{h_{21}}$	$\frac{-1}{h_{21}}$	$\mathbf{a_{21}}$	$\mathbf{a_{22}}$

NB $\Delta_z = z_{11}z_{22} - z_{12}z_{21}$, etc.

Scattering Parameters

S-parameters treat the signal in terms of voltage waves for a reference impedance - the "system impedance" which is normally 50 Ω.

The four S-parameters of a two-port represent the following transfer of power:-

S_{11}: Reflected back from the input
S_{21}: Forward from the input to the output
S_{12}: Reverse from the output to the input
S_{22}: Reflected back from the output

Since the proportion of signal reflected back from (e.g.) the input depends on the input impedance on the two-port, so the S_{11} is closely related to the input impedance. Three special cases are:

Zero input impedance (**short circuit**): $\mathbf{S_{11}=-1}$ because all the signal is reflected, and the reflected voltage wave must be anti-phase in order to make the voltage at the short circuit zero.

Infinite input impedance (**open circuit**): $\mathbf{S_{11}=+1}$ because all the signal power is reflected, and the voltage is a maximum at the open circuit.

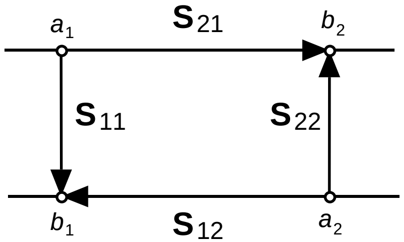
A **matched** input: This means that the input impedance is equal to the system impedance, and leads to $\mathbf{S_{11}=0}$ (no reflected signal, all the power goes into the network).

There is a clear relationship then between S_{11} and reflection coefficient, but we need to be precise....

DEFINITIONS

In the flowgraph below, a_1 is the voltage wave coming into port1 and b_1 is the voltage wave coming out from port1.

a_2 is the wave going into port 2, and b_2 is the wave coming out of port2.



It is very important to recognise that this is a flowgraph, not a circuit diagram. The nodes are not the same as the nodes in a circuit, which were used in AHYZ parameters earlier.

a_1, a_2, b_1, b_2 are called NODES but they represent the voltage waves exactly at the input and output ports. This is important because as soon as you move away from the network's ports the voltage waves change in phase: S-parameters must always have "reference planes" defined as a result.

$S_{11,21,12,22}$ are BRANCHES and represent the signal flow.

From the flowgraph, the value of any node can be calculated by:-

$\sum$ (the value of the branch X the value of the node at the *other end* of that branch)
all branches
pointing *into* the node

Applying this gives:-

$$\mathbf{b_1 = S_{11} \cdot a_1 + S_{12} \cdot a_2}$$

$$\mathbf{b_2 = S_{21} \cdot a_1 + S_{22} \cdot a_2}$$

Each signal coming out of the two port can be seen to have two components: Some signal is transferred from the other port and some is reflected from the port:

Turning these equations around yields the definitions of the S-parameters for a two-port network:-

$S_{11}=b_1/a_1$ (when $a_2=0$)

$S_{22}=b_2/a_2$ (when $a_1=0$)

$S_{21}=b_2/a_1$ (when $a_2=0$)

$S_{12}=b_1/a_2$ (when $a_1=0$)

S-PARAMETER EXAMPLE

A filter is a useful example to give an insight into S-parameter values and the wave behaviour of circuits. First, the deciBel units need to be understood.

dB dB dB dB dB dB dB dB dB dB dB dB

- dB is a unit based on the logarithmic ratio of powers. dB units are able to conveniently represent a huge range of signal levels and high gain/attenuation ratio: e.g. from pW to 100's of Watts is often encountered in radio.

$$\text{Gain in dB} = 10 \log \frac{\text{OUTPUT POWER}}{\text{INPUT POWER}}$$, for an amplifier for example

Note that this is a ratio and has no absolute units.

Since power = V^2/R, it is important to remember to use 20, not 10 log for voltages:-

$$\text{Gain in dB} = 20 \log \frac{\text{OUTPUT VOLTAGE}}{\text{INPUT VOLTAGE}}$$ p-p, rms, or amplitude; it's a ratio

S-parameters in decibels

Because S-parameters are voltage-based, to get dB, you must use:
20 log (magnitude of Snm).

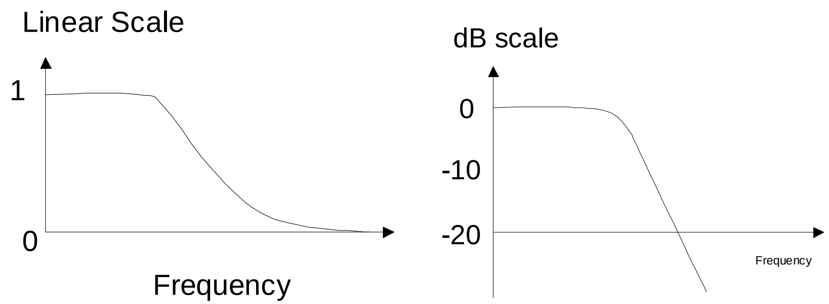
For example, the power gain in dB of an amplifier is **20 log (mag S_{21})**
The return loss is a figure of matching quality and is **20 log (mag S_{11})**

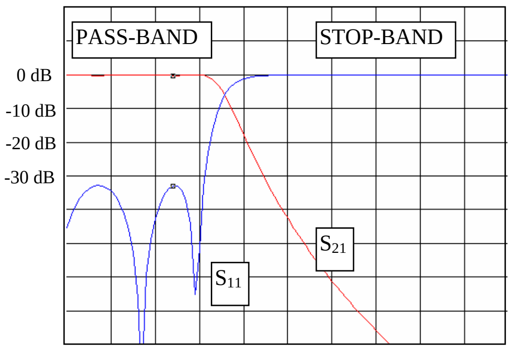
Hence, for the matched case $S_{11}=0$, which gives S_{11}=-infinity dB
For the open or short circuit, mag (S_{11})=1, so mag(S_{11})=0 dB (only the phase is different, and in both the short and open circuit case all of the signal is reflected)

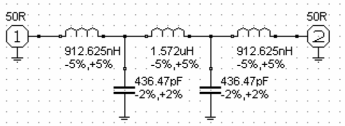
<u>The input or output power must have some absolute units:-</u>

$$Power\ in\ dBW = 10\log(P\ in\ Watts)$$
$$Power\ in\ dBm = 10\log(P\ in\ mW)$$

Compare a LPF in linear units and then in dB:-

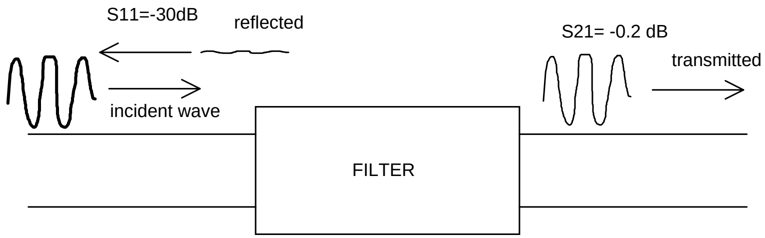




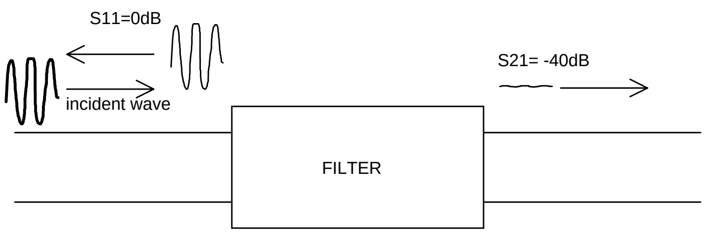


Chebychev Low-Pass Filter Example

IN THE PASS-BAND THIS IS HAPPENING:-



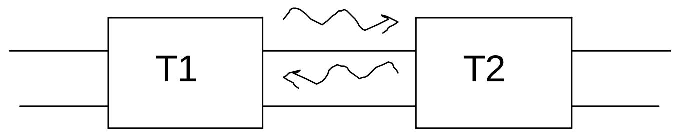
IN THE STOP-BAND THIS IS HAPPENING:-



Transmission parameters

ABCD parameters were previously described as a useful means of dealing with cascaded networks. They deal with the two-port in terms of its terminal voltages and currents. In transmission-line problems it is valuable to have a similar representation which deals instead with the signal in terms of voltage waves, like S-parameters. The parameters that achieve this are called Transmission parameters.
(note: because of the similarity, in some texts you may see ABCD parameters referred to as "voltage-current transmission matrix parameters", and T-parameters as "wave-amplitude transmission matrix parameters").

T-parameters for a cascade of two two-port networks can easily be derived from the S-parameters of each individual network - the output wave from port 2 the first network is set equal to the input wave to port 1 of the second network, and vice versa……..



T = T1 x T2

The T-parameter matrix can be derived from the S-parameters as follows:-

$$T = \begin{bmatrix} 1/S_{12} & -S_{22}/S_{12} \\ S_{11}/S_{12} & \left(S_{12}^2 - S_{11}S_{22}\right)/S_{12} \end{bmatrix}$$