TRANSMISSION LINES & THE SMITH CHART

The purpose of a transmission line is to transfer a signal from a source over some distance to a remote load. In the case of electricity generation and distribution the signal is just a continuous sine wave and the aim is to get a great deal of power from the generator to the user as efficiently as possible. In computer networking, the signal is a pulse train of data. In radio systems the signal is one or more modulated carriers containing the information. The source might be a transmitter in a cabinet on the ground, and the load might be an antenna mounted at the top of a tower.

General transmission line situation

Note that the transmission line has a forward and a return path for current to flow. This is essential in order to obey Kirchoff's laws.

The most important aspects of this transmission line are:-

1. The transmission line must be properly designed to have low loss and be shielded in some way to prevent the signal leaking away and to reject electrical interference from outside.
2. The line has some inductance and capacitance, distributed all the way along the line. This leads to the definition of a CHARACTERISTIC IMPEDANCE (more on this later).
3. There is a **significant time delay** from the signal leaving the source to it reaching the load.

These properties of the transmission line are summarised by the parameters of

Characteristic Impedance, Zo

Propagation Constant, γ. γ has a real part α representing loss and an imaginary part β representing the time delay. $\gamma = \alpha + j\beta$.

Zo is a resistive impedance, but in the ideal lossless case there is no power dissipation associated with it. In circuit terms, as the signal flows down the line the power is temporarily stored in the magnetic field of the line inductance and the electric field of the line capacitance.

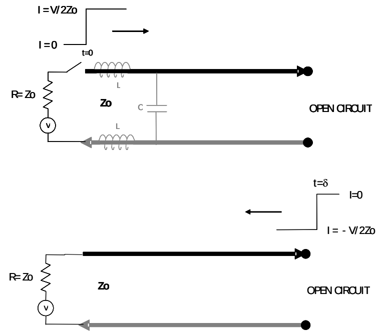
REFLECTIONS ON TRANSMISSION LINES

As a result of the time taken for signals to travel down the transmission line, the source does not at first "know" what the impedance of the load is. If the voltage and current travelling down the line do not match the load impedance, a REFLECTION occurs at the load end. The reflection is created and is essential in order that Ohm's law is obeyed at the load end.

Example 1: Open Circuited Line. A voltage source V with source resistance R is connected by a switch to the transmission line of characteristic impedance Zo at time t=0. To get maximum power from the source into the TL, R is made equal to Zo. The load is an open circuit. Looking at the diagram, it should be obvious that the correct solution is that the current should be ZERO; but we only know that by cheating and looking immediately at the load end. The source can't do that, so initially a current starts to flow at t=0, with value V/2Zo (there is a potential divider effect between the source resistance and the Zo of the TL, giving ½ when R=Zo).

When this current step arrives at the load, the current has nowhere to go, so it is REFLECTED and a reverse step is created at time t=δ, where δ is the time taken to travel down the line.

The value of the reverse step is –V/2Zo, and the two currents cancel out completely, satisfying the expected end result. So, there is some interesting TRANSIENT behaviour, but when everything settles down – in what is know as the "STEADY STATE" - we get the expected result of no current flowing.

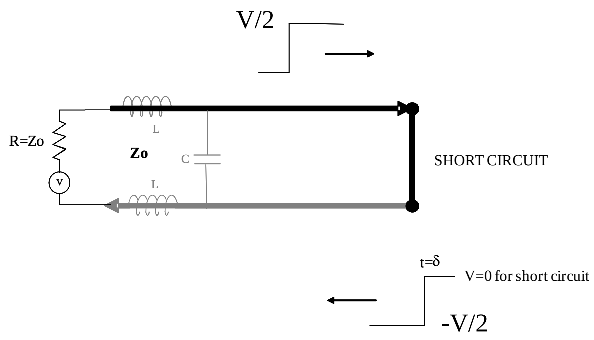


Example 1 of reflections on a transmission line (open circuit line)

Example 2: Short Circuited Line. Now consider what happens when the far end of the line is short circuited. It should be obvious that the voltage at the far end will be ZERO; but we only know that by cheating and looking immediately at the load end. The source does not know what is connected at the end, so initially the voltage step starts to travel down the line, with value V/2.

When this voltage step arrives at the load, Ohm's law is violated, so the step is REFLECTED and a backwards-travelling step is created at time t=δ, where δ is the time taken to travel down the line. The value of the reverse step is –V/2, and the two voltages cancel out at the short circuit end, satisfying Ohm's law.

Reflection coefficient is the ratio of the reflected and incident voltage waves. For the short circuit load, its value is -1, or magnitude 1, phase 180 degrees.



Example 1 of reflections on a transmission line (short circuited line)

In ELECTRICTY POWER TRANSMISSION the transient behaviour can cause huge spikes and destroy equipment.

In COMPUTER NETWORKS the reflections cause data errors, as bits interfere with one another.

In RADIO SYSTEMS the reflections can also lead to damage to components, inefficient transfer of power and data corruption.

The way to avoid these problems is to ensure Zsource=Zload=Zo of the transmission line

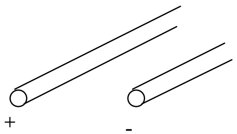
In this case, the reflection coefficient of the matched load is Zero
For the open circuit case, the reflection coefficient is magnitude 1, angle 0 degrees.

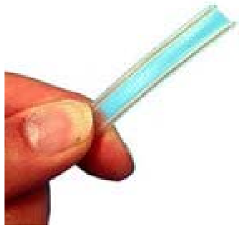
In RF & Microwave Engineering, 50 Ohms is the most commonly used impedance (see measurements notes Appendix)

PRACTICAL CONSTRUCTION OF TRANSMISSION LINES FOR RF & MICROWAVES

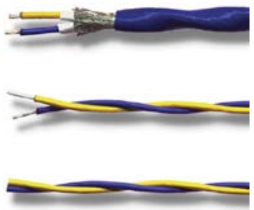
Twisted Pairs and the Lecher Line

Twisted pairs started off life in telephony and were generally regarded as a cheap and simple means of achieving signal confinement for a low frequency transmission line. Nowadays they find widespread use in computer networking: "UTP" stands for Unshielded Twisted Pair and these cables are routinely used to 100Mb/s.





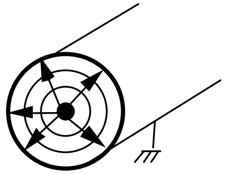
300 Ohm FM ribbon from Waters and Stanton plc.
http://www.wsplc.com

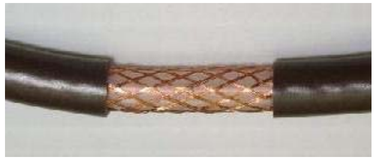


Precision twisted pairs from Gore
http://www.gore.com

Coax

Coaxial lines consist of a centre conductor inside a cylindrical outer ground shield. Standard coax (like in the Teaching labs) is useable to a few hundred MHz. But higher precision types are useable up to 110 GHz.

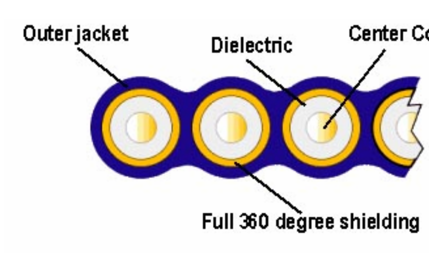




Braided flexible coax

"SMA" connector

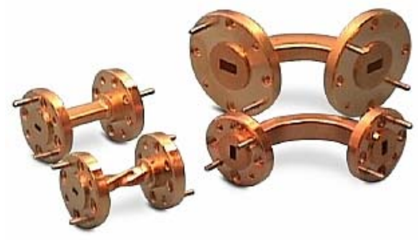
As computers are operating beyond 3GHz, and high data rate connections are required without unwanted reflections, "Micro-coax" has started to make an appearance in ribbon cable form.



"Blue Ribbon" Micro Coax from Tyco Electronics
http://www.precisionint.com/HighSpeedData/BlueRibbon/

Hollow Waveguide

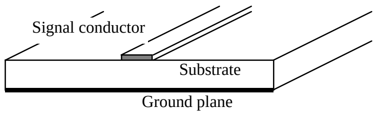
This is no longer a "two-wire" type of transmission line and we can't simply talk about forward and return current paths. The signal propagates as an electromagnetic wave, with a complicated field pattern. They are low loss and can handle high power.



Some rectangular waveguide components with flanges (from www.Quinstar.com)

Microstrip

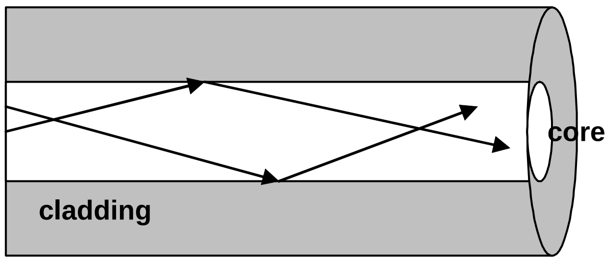
This consists of a signal conductor mounted above a ground plane, usually by using a dielectric substrate. With special materials and precision fabrication, microstrip is useable to more than 100 GHz. On basic "PCB-like" materials it works to maybe a few GHz. It is fabricated with photomasks, so yields cheap components. However, losses are quite high giving moderate performance.



The Satellite TV LNB was an excellent example of a microstrip circuit.

Optical Fibres

These are more and more common in modern communications because of their enormous bandwidth and therefore information-carrying capacity. They are, of course, based on total internal reflection:-

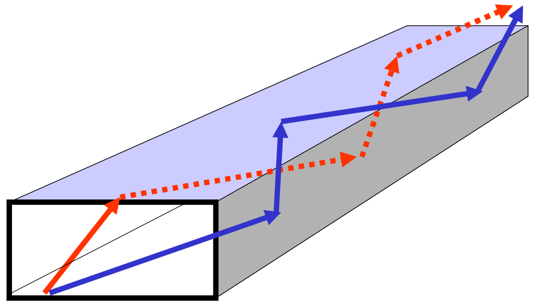


Note that multiple rays can travel down the fibre, and the fibre must ideally be designed in such a way that these rays all have the same propagation time down the fibre and they all arrive in-phase. Without this, data will be corrupted. Making sure the rays all superimpose properly is more properly referred to as designing the fibre to carry the correct *mode of propagation*. If the core is wide in diameter, rays can go off in all directions and many modes can exist. Most long-haul fibres have very narrow cores to ensure that only one mode can exist.

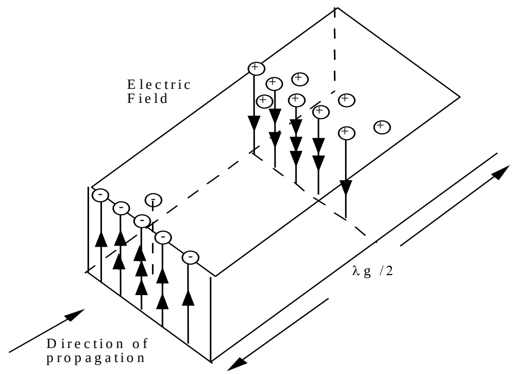
Fibres are key to the backbone of computer networks. They are also useful at this point for explaining the very difficulty subject of metallic microwave rectangular waveguides and their modes of propagation.

RECTANGULAR WAVEGUIDES

These guide the signal by reflections off the metallic sidewalls. A rectangular waveguide is able to support a theoretically infinite number of modes. The one normally used is the TE10 mode, which is the one which starts to propagate at the lowest frequency. This can be visualised as the superposition of two em waves, bouncing back and forth down the waveguide at a critical angle such that they add constructively at the far end:-



The terms TE and TM stand for Transverse Electric and Transverse Magnetic. The mode indices (1,0 etc) in this case refer to the number of half cycles that fit across the waveguide in the *x* and *y* directions. z is the direction of propagation, along the longitudinal axis of the waveguide. In the TE10 mode, the E-field is always perpendicular to the direction of propagation, as illustrated below:-



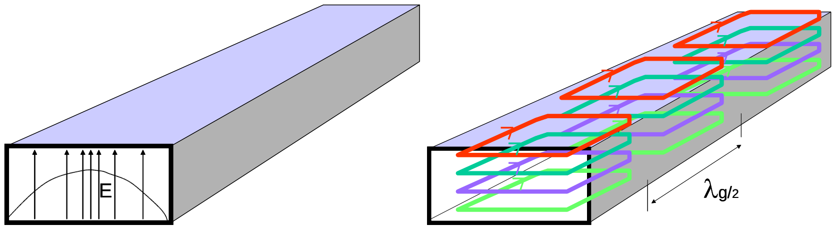


Illustration of the E-field (left) and H-field (right) for the TE10 waveguide mode

CUT-OFF FREQUENCY

Observing the E-field distribution from the previous page, it is necessary for a half-wavelength of the signal to be “fitted inside” the guide. This means that there is a LOWER CUT-OFF for the TE10 mode given by:-

$$a \le \frac{\lambda_{gc}}{2} \rightarrow f_c = \frac{V}{2a}$$

$$f_c = \frac{C}{2a\sqrt{\mu_r \varepsilon_r}}$$

Below this frequency, power cannot flow and the incoming wave is Evanescent

→ exponential decay with distance

→ only reactive power.

There is also an upper frequency limit because higher order modes can be excited; TM_{11}, TE_{20}, etc. Multiple modes = bad news. The mode velocities are different, so an information-carrying signal suffers massive pulse spreading (this is called DISPERSION).

Because of this multiple-mode characteristic, rectangular waveguides have a restricted useful bandwidth, and so a range of standard sizes is used to cover the whole microwave and millimetre-wave spectrum:-

FREQUENCY BANDS

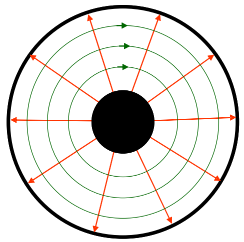
L	1	to	2
S	2	to	4
C	4	to	8
X	8	to	12
Ku	12	to	18
K	18	to	26.5
Ka	26.5	to	40
Q	33	to	50
U	40	to	60
V	50	to	75
W	75	to	110

> 1 GHz = microwave
> 30 GHz = mm waves or “EHF”
> 300 GHz = submm-waves…….and after that you move into infrared!

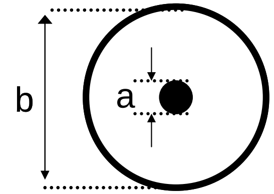
COAXIAL LINES

Analysis of the electric and magnetic fields in the coaxial structure is reasonably straightforward because of its simple structure. The field is referred to as TEM (Transverse Electromagnetic) because the E and H field lines are both perpendicular to the direction of propagation. The H-field forms circular loops around the current in the centre conductor. The E-field forms radial lines between the centre conductor and the grounded outer conductor. The field **pattern** (not strength though) is exactly the same all the way down the cable.

Analysis yields the following characteristic impedance:-



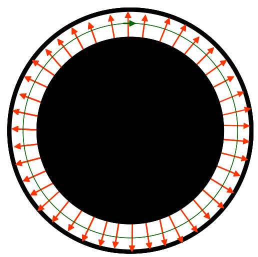
$$Z_o = \sqrt{\frac{L}{C}} = \frac{377}{2\pi\sqrt{\varepsilon_r}} \ln\left(\frac{b}{a}\right)$$

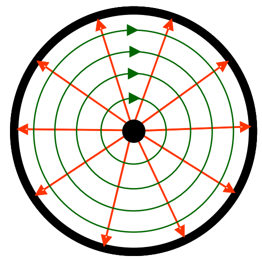


L and C are distributed along the cable. This is a vital concept in high frequency engineering that will be examined in detail later.

COAXIAL ‘CIRCUITS’

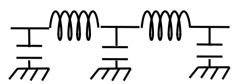
At low frequency, a high impedance line section will appear as a series inductance, whilst a low impedance section will act as a shunt capacitance:-

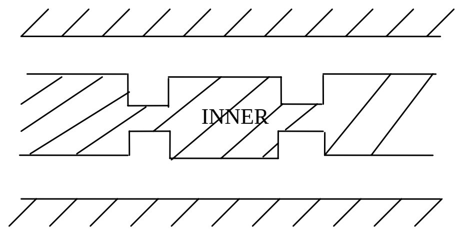

Low Z


High Z

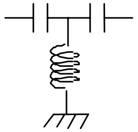
So, a low pass filter can be realised with a machined centre conductor:-

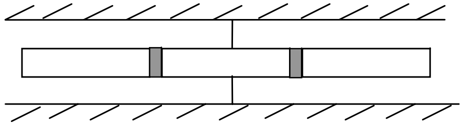
FILTERS:-



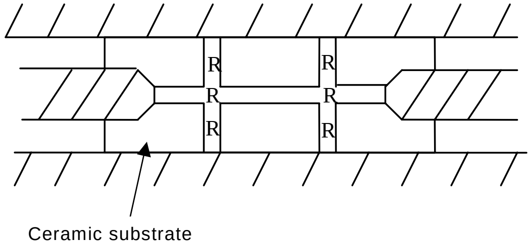


A **thin** disc shorting the centre conductor will act as a shunt inductor. A dielectric spacer in series in the centre conductor will act as a series capacitance. Hence, a high-pass filter can be realised:-





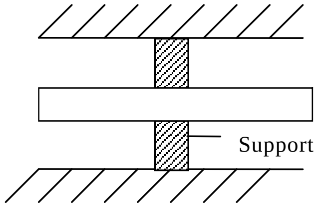
More complex components can be realised by inserting a ceramic substrate into the coax, with resistors printed on it, or even devices attached. The example shown below is an attenuator. Power sensors would use a similar structure, with a diode detector mounted on the substrate.



Cross-section of a coaxial attenuator

Frequency limitations of coaxial cables and connectors

Connectors with very narrow outer conductor diameter and air instead of dielectric give the highest frequency of operation. But even an air-based line must have supports:-



Poor ground continuity affects low frequency connectors like BNC ones, but even high quality microwave and millimetre-wave connectors and cables are limited in frequency range because of ***non TEM modes*** such as a transverse resonance in the support material, or transverse resonance in the air cavity. If the connector diameter is smaller, these resonances will be at higher frequencies. Hence, to achieve a higher maximum frequency, the inner diameter of the outer conductor must be made smaller. For example, to operate to 110GHz the 1 mm connector was developed in the mid 1990s.

COMMON COAX CONNECTOR TYPES

BNC < 1 GHz
'N' < 18 GHz
APC2.4 < 50
'K' < 40 GHz
"W" , 1 mm <110GHz

APC7 < 18 GHz
SMA < 18 GHz
APC3.5 < 26.5
'V' < 65

APC3.5 is compatible with SMA and K
APC2.4 is compatible with V

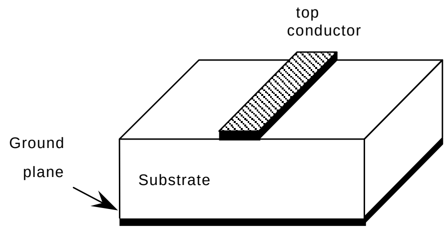
MALE/FEMALE is determined by centre conductor. (In the US "plugs" and "jacks" are often referred too)

MICROSTRIP

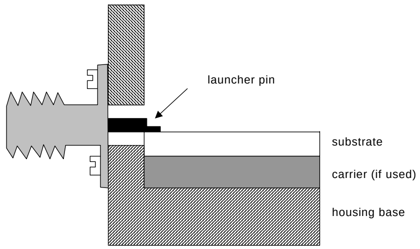
This is a simple low-cost technique for realising microwave transmission lines.

'Microstrip' is the most common for circuits because:-

- It's easy to make
- You can solder on components
- You can make filters, couplers and matching networks simply by 'drawing' them.



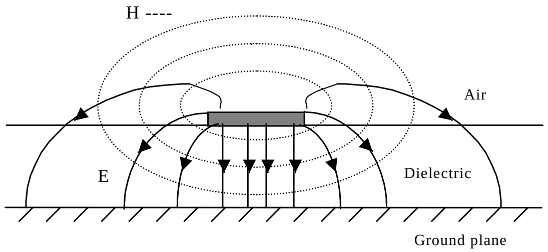
Microstrip



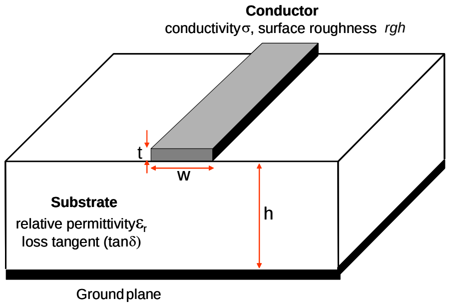
Typical coax-microstrip launcher cross-section

MICROSTRIP ANALYSIS AND DESIGN

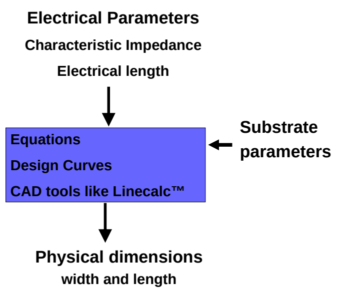
The approximate field pattern is shown below. Since the dielectric is inhomogeneous, it does not support pure TEM fields, but quasi-TEM. This makes it very difficult to analyse directly.



The most basic function of CAD packages for microstrip design is to calculate the electrical parameters for a line of given dimensions:-



Analysis mode; calculate electrical parameters from dimensions & substrate parameters



Synthesis mode

Tools such as “Linecalc” perform both analysis and synthesis. Some care has to be exercised because the methods used may not work for:-

1. Data outside a certain range.
2. Higher order modes of propagation (non quasi-TEM).

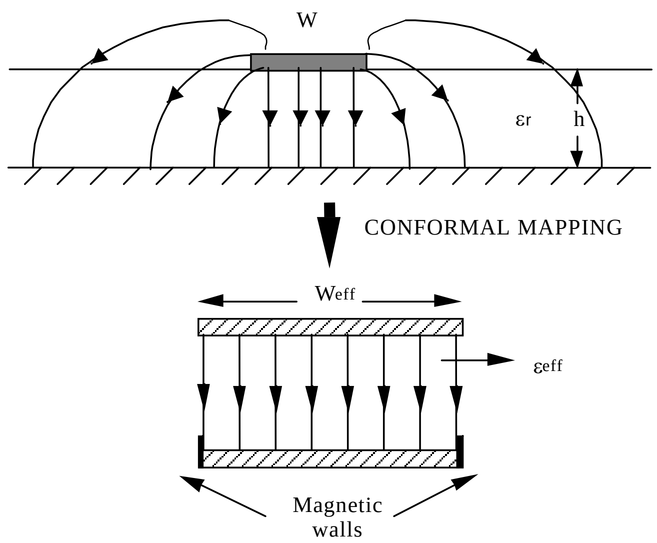
THE ‘MAGNETIC WALL’ MODEL OF MICROSTRIP

The fields in microstrip are too complicated to solve for directly. A common technique is to use “conformal mapping” to map the structure into a parallel-plate waveguide. This is assumed to have no fringeing fields at the edge – they are “magnetic walls”. Thus the field pattern is obvious. By reverse-mapping these field lines into the original microstrip domain, the microstrip fields are found.

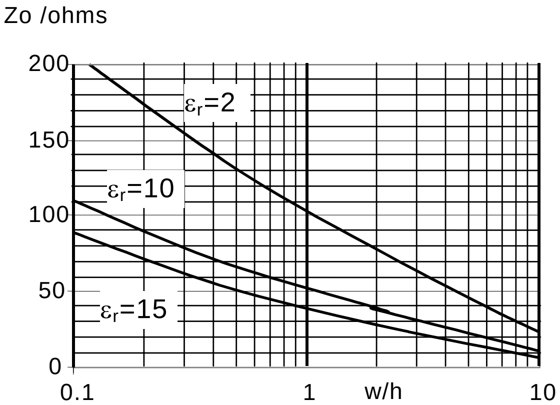
From the effective width and effective dielectric constant, the characteristic impedance can be calculated:-

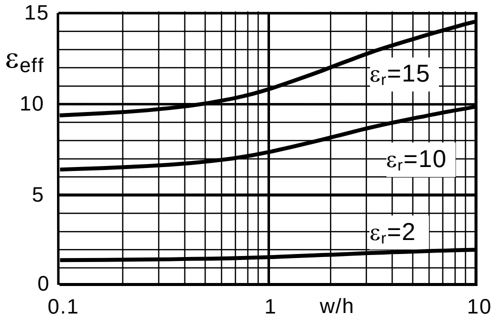
$$Z_0(f) = \frac{h\eta}{W_{eff}(f)\sqrt{\varepsilon_{eff}(f)}}$$

Z_0 changes slightly with frequency.



This kind of analysis yields graphs of Zo and Effective Dielectric Constant vs. w/h ratio for various substrate dielectric constants:- e.g.



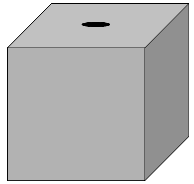


Note: these graphs are not intended for real designs,

TRANSMISSION-LINE RESONATORS

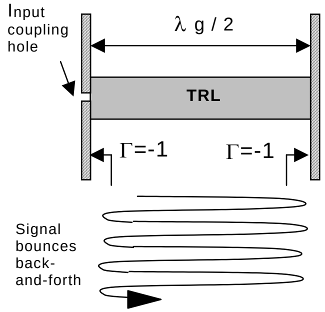
Broadly speaking transmission-line resonators are either based on reflection between a pair of open and/or short circuits, in a straight resonator, OR a circulating signal in a round resonator.

©ImagExtra



Waveguide Cavity

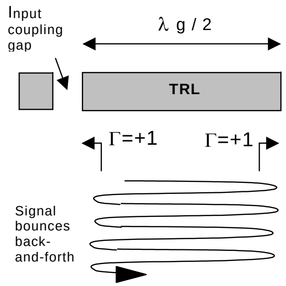
The principle is the same but the bottle resonance is from sound waves reflecting back-and-forth whereas the waveguide cavity resonance is due to electromagnetic waves reflecting back-and-for the from the metallic walls:



e.g. waveguide cavity

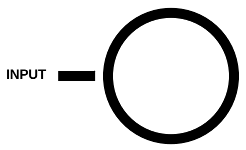
HALF-WAVELENGTH RESONATOR WITH SHORT CIRCUIT ENDS

This resonator has resonant frequencies given by $n\lambda g_{/2}$= physical length where n is an integer



HALF-WAVELENGTH RESONATOR WITH OPEN CIRCUIT ENDS

This resonator has resonant frequencies given by $n\lambda g_{/2}$= physical length where n is an integer.This is widely used in microstrip filters, since short-circuit resonators are hard to fabricate (holes need to be drilled in the substrate to connect to the ground plane).



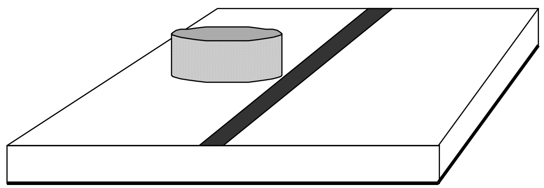
e.g. microstrip ring

CIRCULAR RESONATOR

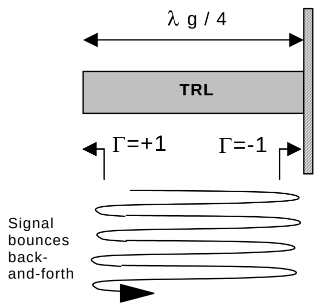
This resonator has resonant frequencies given by $n\lambda g = 2\pi$ x radius where n is an integer.

i.e. the signal circulating round must travel an electrical distrance n x 360 degrees each time around in order for it to add constructively.

Another important example is the DIELECTRIC PUCK; here, the reflection at each side is caused by total internal reflection at the boundary between the dielectric (which has high εr) and air. The cylindrical puck actually has complex field patterns like a hollow waveguide.



A dielectric puck, coupled to a microstrip line



e.g. combline filter

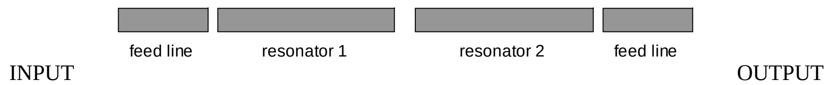
QUARTER-WAVELENGTH RESONATOR; one end open, the other short circuit

This resonator has resonant frequencies given by $n\lambda g_{/4}$= physical length where n is an integer

In all cases, the signal is coupled in and out through something like a hole in a ground plane, or a gap between signal conductors. It is important that the LOADED Q is not lowered too much by

overcoupling. Ultimately, the loss in the transmission line determines the highest Q-factor that can be achieved.

A complete bandpass filter can be constructed by coupling a number of resonators together; e.g.:-

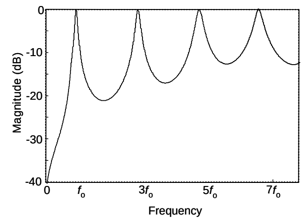


Bandpass Filter using End-Coupled Half-Wavelength Open-Circuited Resonators

Note that this consists only of capacitively-coupled resonators, and cannot be designed directly with the simple synthesis method shown earlier for LC filters.

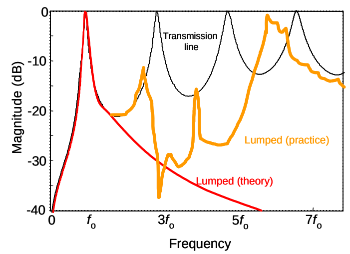
Filter Example: A 10 GHz bandpass filter on alumina (12 x 12 mm)

The resonators are half-wavelength long and open at each end.
They are coupled together side by side.



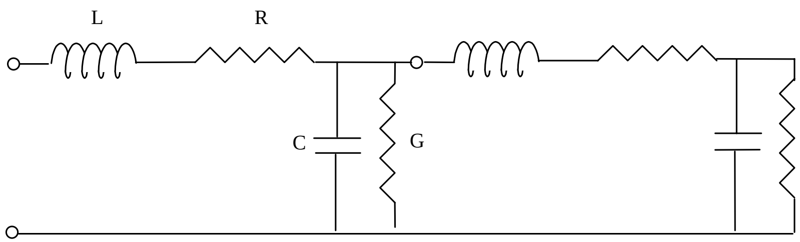
Frequency response of open/short quarter-wave resonator

Swindled? The whole point of replacing lumped elements was to get proper high frequency behaviour. The key difference of distributed elements is that this harmonic behaviour is extremely predictable, whereas lumped element behaviour is almost random at high frequencies:-



Transmission Line Equations

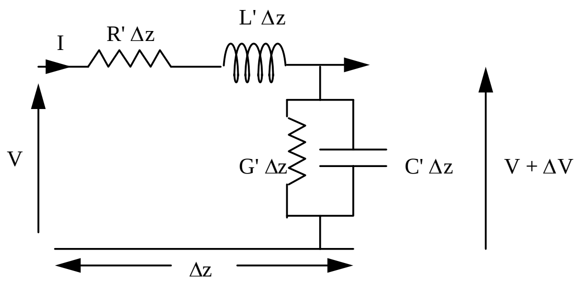
First consider one of the TEM transmission lines (e.g. microstrip) as an LC ladder:-



The energy in the H-field is represented by L.
The energy in the E-field is represented by C.

THIS WORKS IF EACH LINE SEGMENT IS SO SHORT THAT THE E/H PHASES DON'T CHANGE SIGNIFICANTLY. IN PRACTICE THIS USUALLY MEANS ONE-TWENTIETH OF A WAVELENGTH.

TRANSMISSION LINE EQUATIONS are found if we make this model a proper representation of the truly distributed nature by letting the LC section tend towards zero length:-



$$V = (R'\Delta z)I + (L'\Delta z)\frac{\partial I}{\partial t} + (V + \Delta V)$$

$$I = (G'\Delta z)(V + \Delta V) + C'\Delta z\frac{\partial}{\partial t}(V + \Delta V) + (I + \Delta I)$$

as $\Delta z \rightarrow 0$, $V + \Delta V \rightarrow V$

$$-\frac{\partial V}{\partial z} = R'I + L'\frac{\partial I}{\partial t} \quad : \quad \frac{\partial}{\partial z} \quad -\frac{\partial^2 V}{\partial z^2} = R'\frac{\partial I}{\partial z} + L'\frac{\partial^2 I}{\partial z \partial t}$$

and $$-\frac{\partial I}{\partial z} = G'V + C'\frac{\partial V}{\partial t} \quad : \quad \frac{\partial}{\partial t} \quad -\frac{\partial^2 I}{\partial t \partial z} = G'\frac{\partial V}{\partial t} + C'\frac{\partial^2 v}{\partial t^2}$$

$$-\frac{\partial^2 V}{\partial z^2} = R'\left(-G'V - C'\frac{\partial V}{\partial t}\right) + L'\left(-G'\frac{\partial V}{\partial t} - C'\frac{\partial^2 V}{\partial t^2}\right)$$

$$\frac{\partial^2 V}{\partial z^2} = R'G'V + (R'C' + L'G')\frac{\partial V}{\partial t} + L'C'\frac{\partial^2 V}{\partial t^2}$$

Similarly:-

$$\frac{\partial^2 I}{\partial z^2} = R'G'I + (R'C' + G'L')\frac{\partial I}{\partial t} + L'C'\frac{\partial^2 I}{\partial t^2}$$

These are known as the Telegrapher's Equations.

For the LOSSLESS LINE they can be reduced to:-

$$\frac{\partial^2 V}{\partial z^2} = L'C'\frac{\partial^2 V}{\partial t^2} \quad \text{and} \quad \frac{\partial^2 I}{\partial z^2} = L'C'\frac{\partial^2 I}{\partial t^2}$$

Wave Equation Solution:-
The general solution is a pair of waves, one travelling in the positive, and one in the negative direction:

$$V_Z = V_{o+}e^{-j\beta z} + V_{o-}e^{+j\beta z}$$

Where, β = propagation phase constant = ω/V

And the velocity of the wave, $V = \frac{1}{\sqrt{L'C'}}$

$\frac{V_z}{I_z} = Z_0 = \sqrt{\frac{L'}{C'}}$, is the Characteristic Impedance of the line for the LOSSLESS case

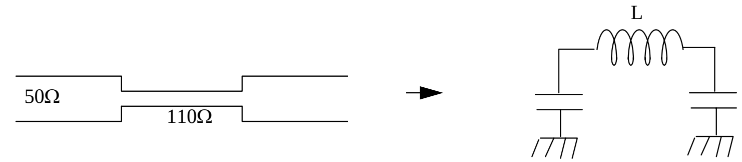
N.B. THE WAVE IS GUIDED. THUS ITS SPEED IS REDUCED BY THE DIELECTRIC:-

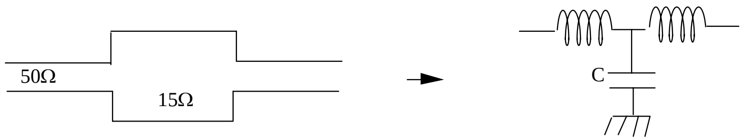
$$\lambda_{guided} = \frac{\lambda_{freespace}}{\sqrt{\varepsilon_r}}$$

For microstrip we use ε_{eff}, which is a combination of air (1) and the substrate dielectric relative permittivity.

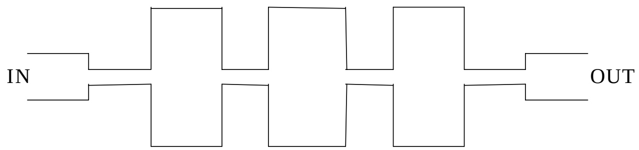
$\omega = 2\pi f$ $V = f\lambda_g$ for a guided wave $C = f\lambda_0$ in free space

The L-C equivalence can be used to make filters and matching networks:-





Thus, a microstrip lowpass filter is easily realised by connecting these series-L and shunt-C sections of high-impedance and low-impedance line:-

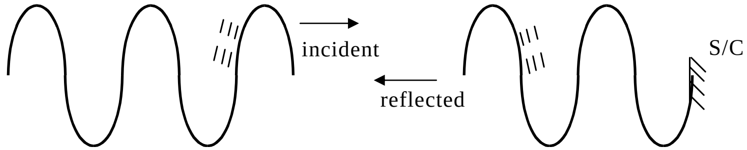


Stepped-impedance lowpass filter

STANDING WAVES AND THE SMITH CHART – distributed circuits

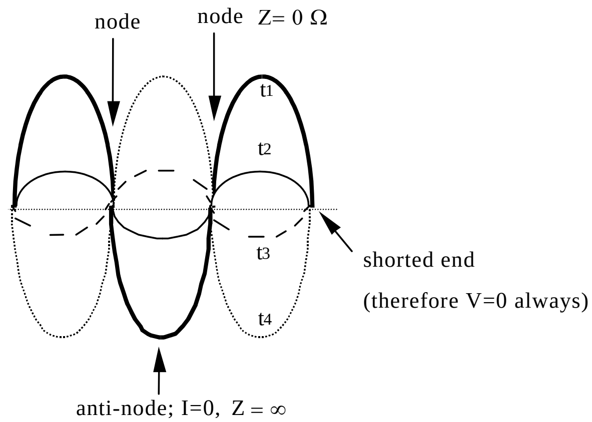
The simple microstrip lowpass filter demonstrates how conveniently a high frequency circuit can be realised. However, in order to fully understand transmission-line circuits it is necessary to learn about standing waves, and about how impedance is transformed as you move along a transmission line.

Consider a short-circuited transmission line: clearly all the incident signal is reflected: there is still a continuous incident signal, however:-



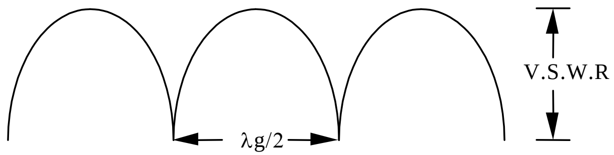
This is the classic interference problem from sound, optics etc., etc.

You get a STANDING WAVE with constructive and destructive interference at different points in the transmission line. The reflected wave takes a phase that ensures that V=0 at the short circuit.



If a diode detector is moved along the transmission line, the envelope of the standing wave will be traced out. The Voltage Standing Wave Ration (VSWR) is the ratio of the maximum and minimum voltage along the line. In the case of the short circuit, VSWR=∞ since the nodes have V=0. In the steady state there is no power flow in the transmission line.

With a proper termination there is no reflected power and no standing wave, VSWR=1, and the power travels nicely from the source into the load.



AS YOU MOVE ALONG THE LINE, THE IMPEDANCE CHANGES

This means:-

1. ALL MEASUREMENTS OF IMPEDANCE MUST HAVE A <u>REFERENCE PLANE</u> SPECIFIED.

2. TRANSMISSION LINES CAN BE USED TO <u>TRANSFORM</u> IMPEDANCES FOR MATCHING NETWORKS AND AMPLIFIER DESIGN

<u>Reflection coefficient</u> $\Gamma = \frac{reflected}{incident} = \frac{Z_L - Z_0}{Z_L + Z_0}$ also $Z_L = Z_0\left(\frac{1+\Gamma}{1-\Gamma}\right)$

-1	≤	$\lvert\Gamma\rvert$	≤	+1
complete reflection, antiphase (short cct)				complete reflection inphase (open cct)

<u>Voltage Standing Wave Ratio</u> – VSWR

$VSWR = \frac{V_{max}}{V_{min}}$ $1 \le VSWR \le \infty$

no standing wave — voltage zeros → open OR short cct

$$\lvert\Gamma\rvert = \frac{VSWR-1}{VSWR+1} \qquad VSWR = \frac{1+\lvert\Gamma\rvert}{1-\lvert\Gamma\rvert}$$

The RETURN LOSS in dB is the amount by which the reflected power is less than the incident power which is $= 10\log\frac{P_{incident}}{P_{reflected}} = -20\log\lvert\Gamma\rvert dB$

(20log because reflection coefficient is a ratio of voltage waves)

Many people debate this minus sign. Generally, engineers would say (for example) that an amplifier has "20dB return loss" (i.e. a positive number), which is consistent with the version used here.

IMPEDANCE TRANSFORMATION ALONG A LINE

The impedance changes as you move along the line. To analyse this it is useful to start with the reflection coefficient. The incident wave and reflected wave are travelling in different directions. So, if you move a distance *d* down the line, the new reflection coefficient is given by:-

$\Gamma_{(d)} = \Gamma_L e^{-2j\beta d}$ if its lossless!

Where, β = phase constant = $2\pi / \lambda_g$ radians/metre
d = distance away from the load metres.
Γ_L is the reflection coefficient at the load

Note that βd is the **<u>ELECTRICAL LENGTH</u>** that you have moved along. The reflection coefficient phase changes byt TWICE that value, because both the incident and reflected waves change phase as you move.

The <u>impedance</u> at any point on the line is given by:-

$$Z_{(d)} = Z_0\frac{1+\Gamma_{(d)}}{1-\Gamma_{(d)}} = Z_0\frac{1+\Gamma_L e^{-2j\beta d}}{1-\Gamma_L e^{-2j\beta d}}$$

$$C = f\lambda_0 \qquad V = f\lambda_g \qquad \lambda_g = \frac{\lambda_0}{\sqrt{\varepsilon r}} \text{ or } \lambda_g = \frac{\lambda_0}{\sqrt{\varepsilon eff}}$$

Using $\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}, Z_{(d)} = \frac{Z_0(Z_L+Z_0)+(Z_L-Z_0)e^{-2j\beta d}}{(Z_L+Z_0)-(Z_L-Z_0)e^{-2j\beta d}}$

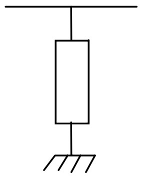
This gives the important result

$$Z_{(d)} = Z_0\frac{Z_L + jZ_0\tan\beta d}{Z_0 + jZ_L\tan\beta d}$$

and similarly it can be shown that $Y_{(d)} = Y_0\frac{Y_L + jY_0\tan\beta d}{Y_0 + jY_L\tan\beta d}$

EXAMPLE TRANSFORMATIONS

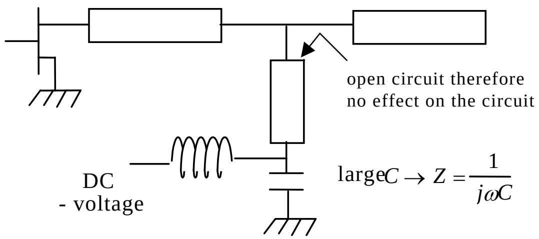
(1) **Quarter-wavelength shortened line.**



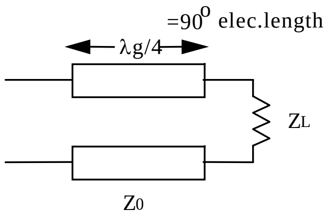
$$\beta = \frac{2\pi}{\lambda_g}, \quad d = \frac{\lambda_g}{4}, \quad Z_L = 0$$

$$Z_{IN} = Z_0 \frac{0 + jZ_0 \tan \frac{2\pi}{\lambda_g} \lambda_g/4}{Z_0 + j0} = j\,\infty \qquad \text{an open circuit}$$

Example use: DC bias injection



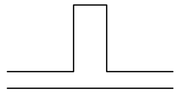
(2) **Quarter-wave series line:**



$$Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan\frac{\pi}{2}}{Z_0 + jZ_L \tan\frac{\pi}{2}} = \frac{Z_0^{\,2}}{Z_L}$$

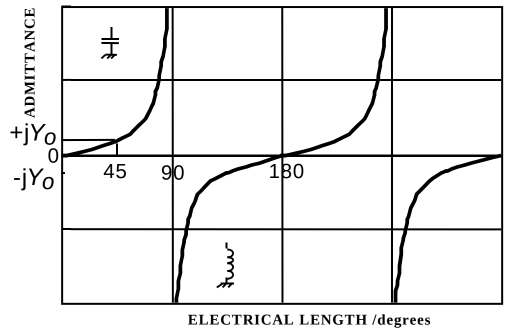
can be used as an impedance transformer

(3) **Open Circuited Stub**



Y_L=0 open cct

$$Y_{IN} = Y_0 \frac{0 + jY_0 \tan \beta d}{Y_0 + j0} = jY_0 \tan \beta d$$



Any value of +/- reactance can be obtained: CHOOSE LENGTH FOR REQUIRED EQUIVALENT L/C VALUE

used for stub matching

TRANSMISSION LINE CALCULATIONS

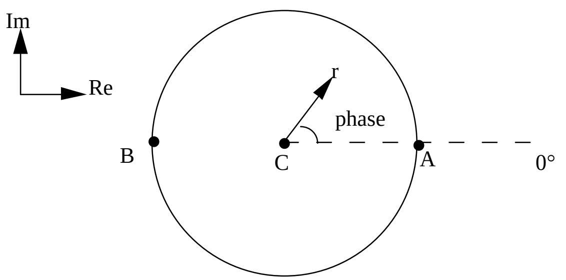
1. MATHEMATICAL: $e^{j\theta}$'s
2. GRAPHICAL
3. COMPUTER

FOR OVER 30 YEARS SPECIAL GRAPHICAL TECHNIQUES HAVE BEEN USED FOR TRANSMISSION LINE PROBLEMS –

THE SMITH CHART

WHILST COMPUTERS ARE NOW EXCELLENT FOR ANALYSING AND OPTIMISING CIRCUITS, THE SMITH CHART IS STILL THE BEST FOR UNDERSTANDING PROBLEMS AND DEVISING SUITABLE MATCHING NETWORKS.

THE SMITH CHART IS A POLAR PLOT OF REFLECTION COEFFICIENT, Γ



OPEN CIRCUIT: ALL SIGNAL REFLECTED, therefore mag. = 1
VOLTAGES MUST ADD IN PHASE
AT ANTI-NODE, therefore → POINT A.

SHORT CIRCUIT: ALL REFLECTED, therefore mag. = 1
but VOLTAGE AT SHORT = 0, therefore
SIGNALS MUST BE ANTI-PHASE:
∅ = 180° → POINT B.

PERFECT LOAD: NO REFLECTION, mag. ≡ 0
→ POINT C.

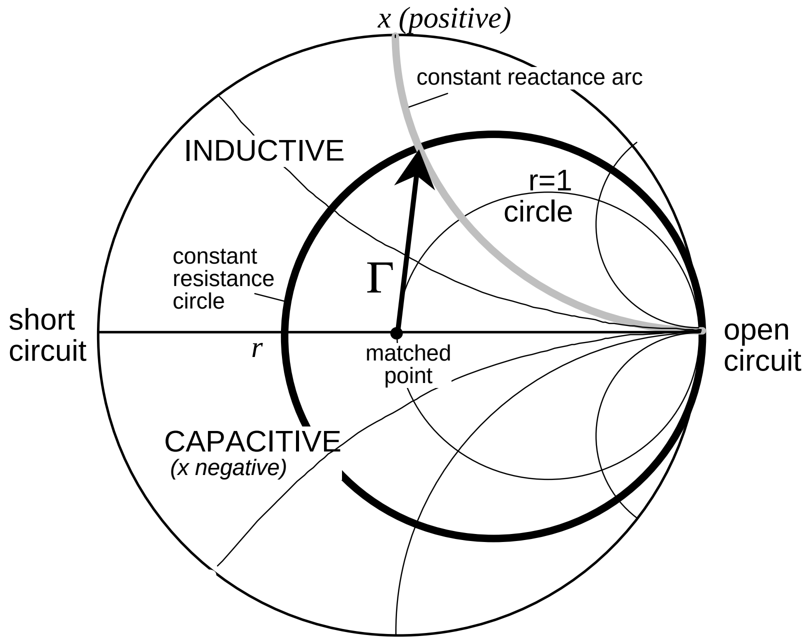
If Γ is plotted for all possible values Z_L (i.e. all possible series resistor/capacitor/inductor combinations), then constant resistance and reactance contours appear. This is discussed later as part of matching network design.

The important feature of the Smith Chart is that you can see how Z varies as you move down the transmission line by rotating the load around the centre point with a compass.

THE SMITH CHART

The Smith chart is a polar diagram on which the reflection coefficient of the load can be conveniently plotted. Since the reflection coefficient has a maximum value of ONE, the diagram is circular with a radius of one. It can be shown that circles of constant load resistance and reactance can be added to the graph : - this makes the chart immediately useful as a calculator for converting from Z to Γ and back again. To achieve this, the contours are all labelled with NORMALISED values of resistance, *r*, and reactance, *x*. Normalising means simply dividing by the system reference impedance Zo.

The key features with an example load are labelled below:-

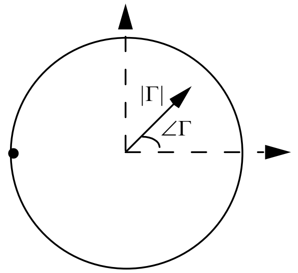


Impedance chart

The key feature that makes the Smith chart so valuable is that since it is based around Γ , it is very easy to cope with transmission-line problems: as you move along a transmission line the magnitude of Γ is fixed, so you are actually rotating around the centre of the chart with fixed radius. By convention, if you move away from the load you rotate clockwise, and vice versa.

Derivation of the SMITH CHART

To illustrate how circles of constant resistance and reactance are formed, first convert to rectangular coordinates form:-



Let $Z_L = R + jX = Z_0(r + jx)$

Then $\Gamma = \dfrac{r + jx - 1}{r + jx + 1}$

$= (r - 1 + jx)(r + 1 - jx)/[(r + 1)^2 + x^2]$

complex conjugate of denominator

$$= \frac{r^2 - 1 + X^2 + j\big((r+1)X - (r-1)X\big)}{(r+1)^2 + X^2}$$

$$= \frac{\left(r^2 - 1 + X^2\right) + j(2X)}{(r+1)^2 + X^2}$$

= U (horizontal) + j V (vertical)

$$U - \frac{r}{r+1} = \frac{\left(\left(r^2 - 1 + x^2\right)(r+1) - r\left((r+1)^2 + x^2\right)\right)}{\left((r+1)^2 + x^2\right)(r+1)}$$

$$= \frac{r^3 - r + rx^2 + r^2 - 1 + x^2 - \left[r^3 + 2r^2 + r + rx^2\right]}{\underbrace{\left((r+1)^2 + x^2\right)(r+1)}_{D}}$$

$$= -\frac{r^2 - 2r - 1 + x^2}{..D.} = -\frac{(r+1)^2 + x^2}{.D..}$$

$$V = \frac{2x}{(r+1)^2 + x^2} = \frac{2x(r+1)}{D}$$

---

therefore

$$\left(U - \frac{r}{r+1}\right)^2 + V^2 = \frac{\left(x^2 - (r+1)^2\right)^2 + 4x^2(r+1)^2}{D^2}$$

$$= \frac{\left(x^2 + (r+1)^2\right)^2}{\left((r+1)^2 + x^2\right)^2 (r+1)^2} = \frac{1}{(r+1)^2}$$

We get the equation of a circle:-

$$\left(U - \frac{r}{r+1}\right)^2 + V^2 = \frac{1}{(r+1)^2}$$

offset on horizontal axis: $\frac{r}{r+1}$; radius $1/(r+1)$

This equation gives a family of constant resistance circles

r=0 gives the edge of the Smith Chart, passing through the short circuit point

As $r \to \infty$, the radius $\to 0$, with the centre point $\to (1,0)$

This is the open circuit point, where the constant resistance circles bunch up and get very small

---

Similarly, it can be shown that:

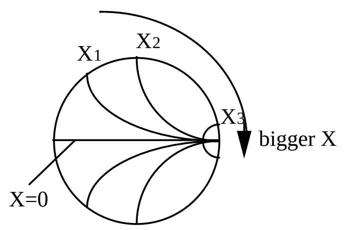
$$(U - 1)^2 + \left(V - \frac{1}{x}\right)^2 = \left(\frac{1}{x}\right)^2$$

Another Circle!

This gives the family of constant reactance contours

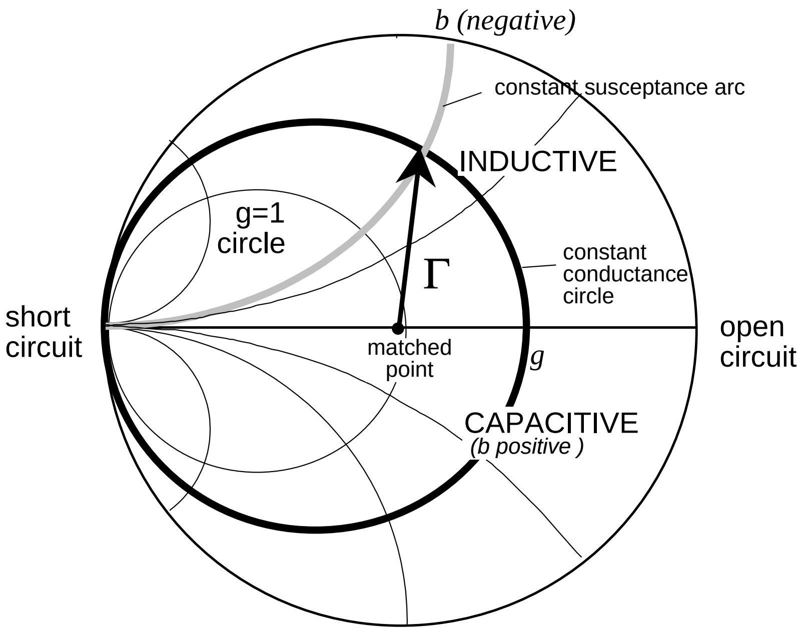
Radius and vertical centre offset both $\to \infty$ as $x \to 0$.

This is the <u>real axis</u> - the only straight line on the chart:-



The admittance chart

The Smith chart can also be used for admittance calculations. The admittance contours are shown below. Note that Γ has not changed, but the contours appear to have flipped over. The admittance chart is vital for shunt stub matching. **UNFORTUNATELY TEXT BOOKS DEAL WITH THE ADMITTANCE CHART IN DIFFERENT WAYS, CAUSING CONFUSION TO MANY STUDENTS.** It is possible to use the same chart (as per the previous page) for both Z and Y, and to convert from one to the other by mirror-imaging Γ through the centre (i.e. rotate 180 degrees). This has the same effect as flipping the contours, BUT the short/open points and all other features are also flipped, causing major confusion. Alternatively, use two charts, or a single chart with both admittance and impedance contours on (if you can handle it).

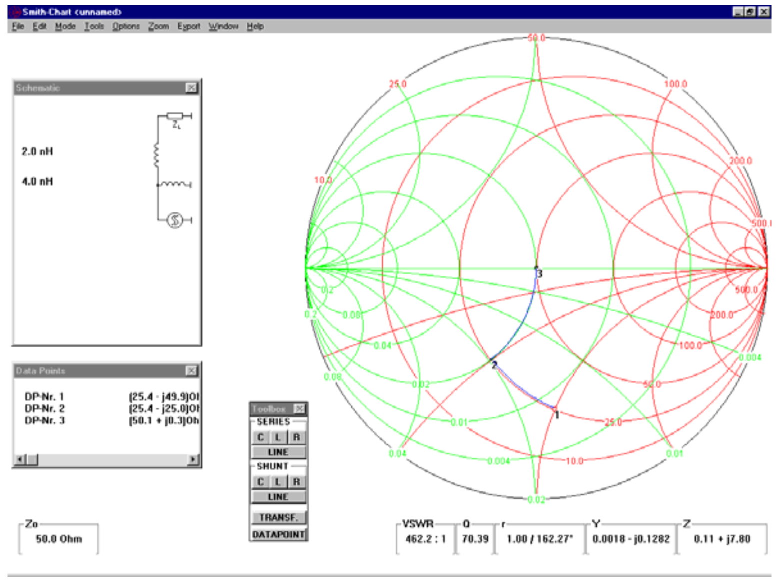


Admittance chart

"SMITH 182" Windows Smith Chart Tool

This is the best free Smith chart tool that I have seen. It plots the impedance and admittance contours in read and green. They can individually be turned on or off. It is available from RF Globalnet.

The example below shows a series/parallel inductor L-network. This will be studied as part of matching and amplifier design.



Screen dump of Smith182 programme showing Z and Y contours simultaneously